

# Toward a Taylor Rule for Fiscal Policy

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## Abstract

In DSGE models, fiscal policy is typically described by simple rules in which tax rates respond to the level of output. We show that there is only weak empirical evidence in favor of such specifications in U.S. data. Instead, the cyclical movements of labor and capital income tax rates are better described by a contemporaneous response to hours worked and investment, respectively. We show that conditioning on these variables is also desirable from a normative perspective as it significantly improves welfare relative to output-based rules.

*Keywords:* Fiscal policy, Bayesian model estimation, Identification, Variable selection

*JEL:* E62, H30, C51

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## 1. Introduction

Recent empirical work provides evidence that there is a systematic contemporaneous response of fiscal policy to the state of the economy (e.g. Taylor, 2000; Auerbach, 2002). DSGE modelers seek to capture this fact by specifying simple fiscal rules where tax rates respond to output (e.g. Leeper et al., 2010; Forni et al., 2009; Traum and Yang, 2011). Nevertheless, the empirical plausibility of output-dependent tax rates has never been conclusively investigated. As we show by estimating a DSGE model with such output-dependent tax rules using U.S. data, it is hard to find evidence in favor of this specification. For this reason, we use the estimated model to investigate, whether conditioning on output is justified from a normative perspective. In particular, we determine which simple tax rules best approximate Ramsey optimal policies. In doing so, we find that the capital income tax rate optimally conditions on investment and that the labor income tax rate optimally conditions on hours worked. Re-estimating the model provides empirical evidence in favor for such a specification and, moreover, provides reaction coefficients which are significantly different from zero. We conclude that empirical researchers should use such tax rule specifications for describing fiscal policy. As we show, the response of output to an unanticipated tax change would otherwise be systematically overestimated.

In a related study, Benigno and Woodford (2006) determine the feedback variables in fiscal feedback rules and show that a complete description of optimal policy delivers complex rules depending on a large number of variables. For the sake of empirical relevance, this paper limits its focus to simple feedback rules rather than a complete description of optimal policy (e.g. Kirsanova et al., 2007). We follow the approach in Schmitt-Grohé and Uribe (2006), who use simple rules which “[...] are defined over a small set of readily available macro indicators and are designed to ensure local uniqueness of the rational expectations equilibrium”. We consider nine such macroeconomic indicators as potential conditioning variables for our simple rules.

The paper also contributes to the literature by proposing a new way of approximating optimal policy using simple linear rules. This is motivated by the observation that the standard approach for constructing optimal simple rules pioneered by Schmitt-Grohé and Uribe (2007) becomes computationally burdensome once a large number of conditioning variables are considered. In our approach we first solve for Ramsey optimal policy in the estimated model. Then, we simulate the model and estimate policy feedback rules using Bayesian methods which incorporate all nine potential variables. We employ the method postulated by Iskrev (2010) to select the variable which influences most the tax rates’ variance at the optimal allocation. Applying this procedure points to using investment for the capital income tax rule and hours for the labor income tax rule. Conditioning on these variables leads to significant improvements in welfare relative to optimal simple rules that condition on output. In particular, it eliminates welfare losses equivalent to more than 0.4% consumption units (relative to Ramsey optimal policy).

The remainder of the paper is organized as follows: Section 2 presents the DSGE model with the tax rules. In Section 3, we estimate the model and show that there is only weak empirical evidence in favor of output in tax rules. Section 4 determines the conditioning variable that is most important for approximating optimal policy. In Section 5, we discuss the relevance of variable selection in fiscal policy rules with respect to welfare, empirical evidence, and policy analysis. The last section concludes.

## 2. The model

We assume that the private sector and the monetary authority can be described by a conventional New Keynesian DSGE model in the succession of Smets and Wouters (2007). The model includes several real frictions: internal habit formation, capital utilization, and investment adjustment costs. It also features two nominal rigidities, one for wages and one for prices. The fiscal policy sector is modeled by wasteful government spending, transfers, and distortionary taxes on capital and wages.

Since the model is well known, we keep the description brief. The maximization problem of households, final goods firms, and intermediate goods firms together with the corresponding first-order conditions can be found in the online appendix. There, we also provide the solution of the steady-state and a complete list of the log-linear equations, which are necessary to solve the model.

### 2.1. Households and firms

The economy features a continuum of households indexed by  $i \in [0, 1]$ . We assume that households are homogeneous with respect to consumption and asset holdings in equilibrium, i.e. households receive the net cash flow from state-contingent securities ( $\iota$ ). Households are heterogeneous with respect to wages and hours worked in equilibrium. Therefore, only labor services  $l(i)$  provided by household  $i$  and wages  $w(i)$  are indexed by  $i$ . Consumption  $c$ , bond holdings  $b$ , and capital  $k$  are not indexed.<sup>1</sup>

Consumers' preferences are characterized by the discount factor  $\beta$ , the inverse of the intertemporal substitution elasticity  $\sigma_c$ , and the inverse of the labor supply elasticity with respect to wages  $\sigma_l$ . The parameter  $h$  measures the internal habit persistence regarding consumption.  $\psi_l$  normalizes the steady-state number of hours worked. Lifetime utility takes the following functional form:

$$E_t \sum_{t=1}^{\infty} \beta^t \left[ \varepsilon_{q,t} \frac{(c_t - hc_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{l_t(i)^{1+\sigma_l}}{1+\sigma_l} \right], \quad (1)$$

where  $\varepsilon_{c,t}$  denotes a shock to the intertemporal choice of consumption. This shock follows a first-order autoregressive process. The flow budget constraint of household  $i$  is given by:

$$\begin{aligned} c_t + I_t + b_t = & (1 - \tau_t^w) \frac{W_t(i)}{P_t} l_t(i) + \left( (1 - \tau_t^k) r_t^k u_t - \phi_t(u_t) \right) k_{t-1} \\ & + \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 - \tau_t^d) d_t + \iota_t(i) + \tau_t^T. \end{aligned} \quad (2)$$

The household invests  $I$  into capital  $k$ . The rental rate on capital is denoted by  $r^k$  and firms' dividends by  $d$ . The household receives transfers  $\tau^T$  and pays distortionary taxes  $\tau^w$  and  $\tau^k$  on labor income and capital income respectively. Households hold government bonds  $b$  yielding nominal risk-free return  $R$ . Additionally, the model features varying utilization of private capital. The cost of varying the intensity of capacity utilization is given by  $\phi(\cdot)$ . The law of motion for private capital is given by

$$k_t = (1 - \delta) k_{t-1} + \varepsilon_{i,t} \left[ 1 - s \left( \frac{I_t}{I_{t-1}} \right) \right] I_t. \quad (3)$$

<sup>1</sup>Throughout the model description, upper-case letters denote nominal variables and lower-case letters real variables. An exception is investment, which is always expressed in real terms as  $I$ .

Capital depreciates at a constant rate  $\delta$  and investments are subject to a convex investment adjustment cost  $s(\cdot)$ . In addition,  $\varepsilon_i$  denotes an investment-specific efficiency shock to the adjustment costs and is supposed to follow a first-order autoregressive process (see Justiniano et al., 2011).

Households maximize lifetime utility (1) subject to the budget constraint (2) and the capital accumulation equation (3). Wage setting is modeled following Erceg et al. (2000), i.e. we assume that monopolistic competitive households set nominal wages in staggered contracts. Each household supplies a differentiated type of labor service,  $l(i)$ . Labor service is aggregated into a homogenous labor good ( $l^d$ ) by a perfectly competitive labor contractor. The labor contractor has access to a Dixit-Stiglitz aggregator production function with  $\theta_w > 1$  denoting the elasticity of substitution. For any wage rate, each household supplies as many labor services as demanded. In each period, household  $i$  is allowed to set its wage with probability  $1 - \gamma_w$ . If the household is not allowed to set its wage, wages are adjusted by the steady-state inflation rate of the economy.

The economy consists of two firm sectors. In one sector, perfectly competitive firms produce the final good  $y$  using as inputs intermediate goods  $y(j)$  produced by monopolistically competitive firms indexed by  $j$ . Final goods firms have access to a Dixit-Stiglitz aggregator production function, with  $\theta_p$  denoting the elasticity of substitution. Cost minimization yields the demand for each intermediate good  $y(j)$  and the corresponding price index.

The intermediate goods are produced by a continuum of monopolistically competitive firms  $j \in [0, 1]$  using the production function

$$y_t(j) = (u_t k_{t-1}(j))^\alpha \left( l_t^d(j) \varepsilon_{z,t} \right)^{1-\alpha} - \Omega, \quad (4)$$

where  $\alpha$  denotes the output elasticity with respect to capital and  $\Omega$  the fixed costs of production. The variable  $\varepsilon_z$  represents a labor-augmenting productivity shock assumed to follow a first-order autoregressive process. Intermediate goods firms are subject to staggered price setting, i.e. they are allowed to adjust their prices with probability  $(1 - \gamma_p)$ . Prices of firms which cannot re-optimize are adjusted by the steady-state inflation rate of the economy.

## 2.2. Government sector and aggregation

The formulation of sticky prices and wages implies inefficiencies and output losses relative to an economy with flexible prices in the goods and labor market. Since we are interested in the optimal allocation of this economy, we have to take the effects of price and wage dispersion into account when aggregating across firms and households. We follow Schmitt-Grohé and Uribe (2006) and capture the resource costs induced by inefficient price dispersion and the resource costs caused by wage dispersion with the auxiliary variables  $p_t^+$  and  $w_t^+$  respectively. The dispersion of wages causes dispersion in utility across households. This dispersion is represented by the variable  $\tilde{w}_t^+$ .

The monetary authority sets nominal interest rates according to the following Taylor rule, with  $\varepsilon^m$  denoting the monetary policy shock:<sup>2</sup>

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (\rho_\pi \hat{\pi}_t + \rho_y \hat{y}_t^m) + \tilde{\varepsilon}_t^m, \quad (5)$$

where  $y^m$  denotes output as measured in the data  $y_t^m = y_t - \phi(u_t) k_{t-1}$ .

The fiscal authority receives tax revenues  $x$  and issues bonds  $b$  to finance government consumption expenditure  $c^g$  and transfers  $\tau^T$ . The government budget constraint therefore reads as:

$$b_t - \frac{R_{t-1} b_{t-1}}{\pi_t} = c_t^g + \tau_t^T - x_t. \quad (6)$$

Government tax revenues consist of taxes on wages and capital:

$$x_t = \tau_t^w w_t \frac{l_t}{w_t^+} + \tau_t^k \left[ y_t - w_t \frac{l_t}{w_t^+} \right]. \quad (7)$$

Government consumption expenditure  $c^g$  and transfers  $\tau^T$  evolve according to exogenous first-order autoregressive processes.

<sup>2</sup>We log-linearize the model around its steady-state. The log-linear deviations of the generic variable  $x$  from its steady-state  $\bar{x}$  is denoted by  $\hat{x}$ .

For the feedback rules of the tax rates, we take the standard rules from the literature as a starting point:

$$\hat{\tau}_t^w = \rho_w \hat{\tau}_{t-1}^w + (1 - \rho_w) (\eta_{wb} \hat{b}_{t-1} + \eta_{wy} \hat{y}_t^m) + \hat{\epsilon}_{t,\tau^w}, \quad (8)$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1 - \rho_k) (\eta_{kb} \hat{b}_{t-1} + \eta_{ky} \hat{y}_t^m) + \hat{\epsilon}_{t,\tau^k}, \quad (9)$$

where  $\hat{\epsilon}_{t,\tau^w}$  and  $\hat{\epsilon}_{t,\tau^k}$  denote *i.i.d.* error terms. We will refer to the model closed with these rules as the baseline model.

### 3. Empirical (ir)relevance of output in tax rules

#### 3.1. Data

We estimate the baseline model using quarterly US time series between 1983:1 and 2008:3. This particular sample was chosen for two reasons: first, to exclude the high-inflation period during the 1970s and the Volcker disinflation years as well as the recent financial crisis, and second, because in this period monetary policy is thought to be active, whereas fiscal policy is assumed to be passive (following Leeper, 1991).

The DSGE model exhibits six structural shocks plus two structural shocks related to the labor and capital income tax rates capturing discretionary changes. Since we want to describe the cyclical movement of the real sector and the fiscal sector as accurately as possible, we add three more shocks to the model. These are an *i.i.d.* shock to the resource constraint, one *i.i.d.* shock to the wage Phillips curve, and one shock following an AR(1) process to the price Phillips curve.<sup>3</sup> Moreover, we add a measurement error to the tax revenue observation equation.

In line with Leeper et al. (2010), who argue that incorporating trends in a model with fiscal policy is non-trivial, because several fiscal variables appear to have their own trends, we do not incorporate one or two trends in the model. However, we want to explain with the model several macro time series which exhibit a trend. In order to match the model with the data, we employ the first-difference filter to the non-stationary time series (output, private consumption, private investment, real wages, government bonds, government consumption, and tax revenues) and remove their individual mean afterwards. We do not allow for a trend in variables, which theory would usually identify as stationary: nominal interest rates, hours worked, inflation, and both tax rates. These time series are demeaned prior to the estimation. Finally, we estimate the model using

$$\left[ \Delta \hat{y}_t^m, \Delta \hat{c}_t, \Delta \hat{I}_t, \hat{l}_t, \Delta \hat{w}_t, \hat{\pi}_t, \hat{R}_t, \hat{\tau}_t^w, \hat{\tau}_t^k, \Delta \hat{b}_t, \Delta \hat{c}_t^g, \Delta \hat{x}_t \right] \quad (10)$$

as the vector of the 12 observable variables, where  $\Delta$  indicates first-differences.<sup>4</sup>

#### 3.2. Estimation

As is common in the literature, we calibrate parts of the parameter vector and estimate the remaining parameters using Bayesian estimation techniques. The calibration and prior choice of the parameters correspond to the related literature.<sup>5</sup> Concerning the government sector, we set the steady-state ratio of government consumption expenditures to output  $\bar{c}^g/\bar{y}$  and the steady-state ratio of transfers to output  $\bar{\tau}^T/\bar{y}$  to 8.5% and 10.5% respectively. This implies an annual debt-to-output ratio of approximately 50%. The steady-state values for the tax rates on capital  $\bar{\tau}^k = 0.1929$  and wages  $\bar{\tau}^w = 0.2088$  are the averages of the corresponding time series.

For the coefficients in front of output in the feedback rules, the literature usually assumes a Gamma distribution, which is strictly positive.<sup>6</sup> We deviate from the standard prior choice for the following reason. The choice of the Gamma distribution is motivated by the estimation results of Blanchard and Perotti (2002), which suggest a positive elasticity between tax revenues and output, also implying a positive elasticity of the total households' average tax rate with output. However, this does not imply that each tax instrument will face a positive elasticity with output.

<sup>3</sup>The latter two shocks are often described in the literature as mark-up shocks. However, since the literature disagrees on whether these shocks have a structural interpretation (Chari et al., 2009) we do not consider these shocks or the *i.i.d.* shock to the budget constraint as structural shocks.

<sup>4</sup>A detailed description of the data source and its transformation can be found in the online appendix.

<sup>5</sup>An overview of the calibrated parameters and of the prior distributions can be found in the online appendix.

<sup>6</sup>This assumption can, for example, be found in recent studies such as Leeper et al. (2010), Traum and Yang (2010), and Fernández-Villaverde et al. (2011).

Therefore, we use a less restrictive and more diffuse prior which is assumed to be Normal-distributed with mean 0 and a standard deviation of 0.5. We employ the same prior distribution for the coefficients on government debt. Moreover, we impose the additional restriction that the solution is non-explosive and unique. In particular, the routine to find the mode of the posterior distribution is initialized within the part of the prior distribution that corresponds to a non-explosive and unique solution of the model.

The estimation results indicate that the posterior distributions of all structural parameters are well approximated around the posterior mode. The results also show that all parameters of the private sector are identified as being substantially different from their prior distribution.<sup>7</sup>

In both tax rules (8) and (9), the autoregressive parameters and the feedback coefficients on debt are estimated as different from zero. Figure 1 plots the prior and posterior distribution of the feedback coefficients in front of output in each tax rate rule. While the feedback coefficient in front of output ( $\eta_{ky}$ ) in the capital income tax feedback rule is not identified, the posterior of the feedback coefficient ( $\eta_{wy}$ ) in the labor income tax feedback rule is estimated as different from the prior distribution. Both feedback coefficients are not significantly different from zero. The 90% posterior probabilities are  $[-0.28, 1.03]$  and  $[-0.76, 0.81]$  for  $\eta_{wy}$  and  $\eta_{ky}$  respectively. This finding corresponds to the experiences reported by Forni et al. (2009) for European data. In general, the authors find positive but not significant feedback parameters on the growth rate of output in the feedback rules. Similarly, Traum and Yang (2011) mention that U.S. post-war data are not informative about a feedback parameter for contemporaneous output in the income tax rates. We conclude from this exercise that there is no empirical support for output in the income tax feedback rules.

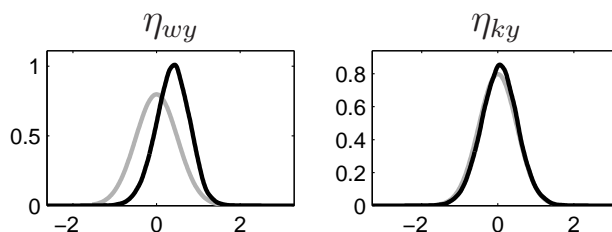


Figure 1: Prior (grey) and posterior (black) distribution of policy feedback parameters.

#### 4. Optimal variable selection in tax rules

Even if there is no empirical support for output in the feedback rules, the choice of output could still be justified from a normative perspective. In this section, we want to investigate whether this is the case or whether different variables should be employed instead. We consider nine readily observable macroeconomic indicators as potential dependent variables. Since we restrict our focus in this paper to simple rules, one possibility would be to employ the method suggested by Schmitt-Grohé and Uribe (2007) and compare different optimized feedback rules. Yet this approach would lead to a high number of potential models, making model comparison very costly in computational terms. We suggest a different approach to selecting the variables in tax rules. In particular, we select those contemporaneous variables which are most important for approximating Ramsey optimal policy with simple rules. To this end, we compute the optimal allocation given the posterior estimates of the model's private sector in Section 4.1. Section 4.2 summarizes the approximation of the optimal policy problem's highly non-linear solution using simple and linear rules. In Section 4.3, we describe how we select among the feedback variables.

##### 4.1. Optimal policy

Based on the structural estimates, we compute the optimal equilibrium of the economy. We assume that the government has operated for an infinite number of periods and honors the commitments it has made in the past. This

<sup>7</sup>To estimate the parameters, we estimate the posterior mode of the distribution and employ a random walk Metropolis-Hastings algorithm to approximate the distribution around the posterior mode. We run two chains, each chain consisting of 1,000,000 parameter vector draws. The first 90% are discarded. A detailed table with the estimation results, prior and posterior posterior plots, along with convergence statistics and further diagnostics, are provided in the online appendix.

kind of policy under commitment is optimal from a timeless perspective (Woodford, 2003). The benevolent policy maker has two instruments, taxes on labor income and taxes on capital income.

The optimal policy problem is defined as maximizing the life-time expected utility subject to the first-order conditions describing the behavior of the model economy.<sup>8</sup> When we compute the optimal policy, we solve for steady-state values of  $\tau^k$  and  $\tau^w$ , which solve the first-order conditions of the policy maker's maximization problem. The steady-states of the tax rates are  $\bar{\tau}^k = -0.1523$  and  $\bar{\tau}^w = 0.3945$ . The implied debt-to-output ratio is  $\frac{\bar{b}}{\bar{y}} = 0.3$ . Since the optimal policy maker in our model faces similar trade-offs as the policy maker in the model by Schmitt-Grohé and Uribe (2006), the steady-state values of the tax rates are similar.

#### 4.2. Approximation of optimal policy rules using linear rules

We want to approximate optimal policy behavior using simple, linear rules. First, we simulate artificial data at the optimal allocation. When simulating the data, we only consider the structural shocks excluding the tax rate shocks ( $\epsilon^i, \epsilon^z, \epsilon^m, \epsilon^q, \epsilon^{cg}$ ) and assume that they exhibit the estimated volatilities.<sup>9</sup>

In our analysis, we want to select between different feedback variables which capture the contemporaneous impact of the state of the economy. Moreover, we follow Schmitt-Grohé and Uribe (2006) and impose the additional condition that the variables have to be easily observable to the policy maker. The conditions determine the two feedback rules for labor income and capital income  $i = [w, k]$  as functions of the following contemporaneous variables:  $\tau_t^i = f(\hat{y}_t, \hat{c}_t, \hat{l}_t, \hat{w}_t, \hat{l}_t, \hat{\pi}_t, \hat{R}_t)$ . We add  $b_{t-1}$  to this set of variables to ensure determinacy and  $k_{t-1}$  as a further stock variable that could be important. The feedback rules for approximating optimal policy behavior thus read:

$$\hat{\tau}_t^w = f(\hat{b}_{t-1}, \hat{k}_{t-1}, \hat{y}_t, \hat{c}_t, \hat{l}_t, \hat{w}_t, \hat{l}_t, \hat{\pi}_t, \hat{R}_t) \text{ and} \quad (11)$$

$$\hat{\tau}_t^k = f(\hat{b}_{t-1}, \hat{k}_{t-1}, \hat{y}_t, \hat{c}_t, \hat{l}_t, \hat{w}_t, \hat{l}_t, \hat{\pi}_t, \hat{R}_t). \quad (12)$$

The corresponding policy coefficients are  $\eta_{ij}$ , where the two subscripts denote the tax instrument ( $i$ ) and their partial elasticities with respect to the feedback variables ( $j$ ) respectively.

To estimate the feedback coefficients, we denote the model economy from Section 2 in the following state space form:

$$\hat{X}_t^o = H\hat{X}_t^z \quad (13)$$

$$\hat{X}_t^z = T(\theta^M)\hat{X}_{t-1}^z + R(\theta^M)X_t^\epsilon. \quad (14)$$

$\theta^M$  is a vector collecting the structural parameters of the model,  $\hat{X}^z$  a vector containing the endogenous state variables,  $\hat{X}^o$  the observable variables, and  $X^\epsilon$  the exogenous variables. We partition the vector into two sub-vectors:  $\theta^M = [\theta^S \ \theta^P]$ . The vector  $\theta^S$  contains all the structural model parameters which are not included in the feedback rules. The coefficients of the fiscal policy rules are included in the vector  $\theta^P$ . Two observable variables are the two income tax rates. When estimating  $\theta^P$  using the artificial data, we need to ensure that the shock processes are correctly recovered and that the volatilities implied by the model, closed with the simple feedback rules, are in line with the volatilities of the artificial data. To do so, we augment the vector of observable variables by three observed shocks ( $\hat{X}_t^o = [\hat{\tau}_t^w \ \hat{\tau}_t^k \ \epsilon_{i,t} \ \epsilon_{i,q} \ \epsilon_{i,cg}]'$ ). In order to ensure that the volatilities implied by the model, closed with the simple feedback rules, are in line with the volatilities of the artificial data, we form the priors for the feedback coefficients endogenously following the method recently proposed by Christiano et al. (2011). As an initial set of priors we define a Normal distribution with mean zero and a standard deviation of one. To check whether the estimated simple linear rules are indeed a good approximation of the optimal policy rules, we compare the implied smoothed time series of the linear tax rules with the time series generated at the optimal allocation. The plots indicate that the simple rules approximate the optimal policy rules satisfactorily.<sup>10</sup>

<sup>8</sup>The maximization problem is formulated in detail in the online appendix.

<sup>9</sup>We do not include the transfer shock in the simulation because it accounts for less than one percent of the variation in any of the variables in the subsequent analysis.

<sup>10</sup>The estimation results of the policy coefficients and the corresponding diagnostics are reported in the online appendix.

### 4.3. Computation of the elasticities and variable selection

Given the estimated coefficients of the large rules (11) and (12), we want to select the most important among them. Our criterion is to ascertain the feedback variables that influence the tax rates' variance at the optimal allocation most. Therefore, we compute the elasticities of the income tax rates' variance with respect to the feedback coefficients. In doing this, we employ the methodology proposed by Iskrev (2010).

Define the vector comprising the variables of interest by  $\hat{X}_t^O = [\hat{\tau}_t^k \quad \hat{\tau}_t^w]'$ . The unconditional variance of  $\hat{X}_t^O$ ,  $\Sigma_{X^O}$ , is calculated from the state space system defined by equations (13) and (14). The matrices  $T$  and  $R$  contain non-linear combinations of the structural parameter vector  $\theta^M$ . In order to take into account the dependence of the variance on the recursive law of motion, which itself depends on structural parameters, the Jacobian  $J(L)$  is decomposed into two Jacobians  $J(L) = J_1 J_2$ , where  $J_1$  contains the partial derivatives of the variance  $\Sigma_{X^O}$  with respect to each recursive law of motion, and  $J_2$  the partial derivatives of each recursive law of motion with respect to each parameter.

We compute partial derivatives with respect to the policy coefficients in  $\theta^P$ .<sup>11</sup> Then, we multiply the partial derivatives by the policy coefficients and divide them by the corresponding moment to calculate the elasticities. To quantify the uncertainty, we take 10,000 draws from the distribution of the policy coefficients derived in Section 4.2. The results are plotted in Figures 2(a) and 2(b). These figures display the box plot of the 75% quantile of the elasticities with respect to each policy coefficient normalized by the largest elasticity.<sup>12</sup> Looking at Figure 2(a), the most important variable to influence the labor income tax rate's variance is hours worked. Figure 2(b) shows that investment is the most important feedback variable for variance in taxes on capital income.

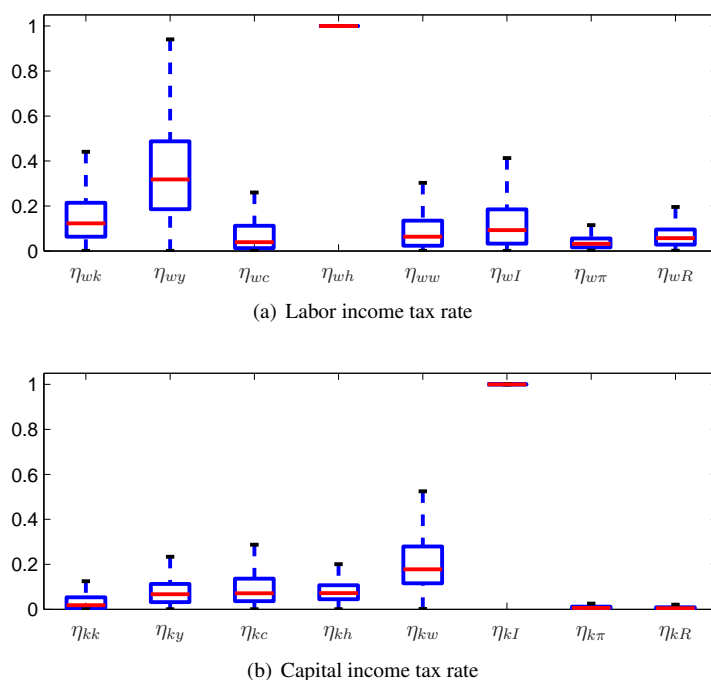


Figure 2: Relative elasticity of variables' variance w.r.t. feedback parameters of the corresponding income tax rule.

We conclude that alongside the missing empirical evidence for output as variable in the feedback rule, output is also not the variable in the feedback rule that has the greatest influence on variance in the income tax rates at the optimal allocation. Instead, different feedback variables are important for different policy instruments. These are the coefficient on hours worked in the rule for labor income taxes as well as the coefficient on investment in the rule for capital income taxes. Both coefficients are also clearly distinguishable from the other feedback coefficients.

<sup>11</sup>Since public debt will be included in any feedback rule to ensure fiscal sustainability, we do not consider it in the subsequent analysis.

<sup>12</sup>The results as well as the related robustness tests are described in more detail in the online appendix.

In addition, we conduct robustness exercises for the variable selection with respect to other potential feedback variables and parameter uncertainty. Firstly, we extend the policy rules (11) and (12) by the variables marginal costs, capacity utilization rate, and the rental rate of capital. While these variables are difficult to observe, they could be important in explaining optimal policy dynamics. Secondly, Schmitt-Grohé and Uribe (2006) point out that price stickiness and investment adjustment costs play an important role in optimal policy. For this reason, we investigate how sensitive our variable selection is in respect to different degrees of investment adjustment costs as well as wage and price stickiness. The results indicate that these parameters affect the volatility and also the optimized coefficient in the simple rules as pointed out by Schmitt-Grohé and Uribe (2006), but they do not affect the variable selection for simple rules. We therefore formulate the following policy feedback rules from a normative point of view:

$$\hat{\tau}_t^w = \rho_w \hat{\tau}_{t-1}^w + (1 - \rho_w) (\eta_{wb} \hat{b}_{t-1} + \eta_{wh} \hat{l}_t) + \epsilon_{t,\tau^w} \quad (15)$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1 - \rho_k) (\eta_{kb} \hat{b}_{t-1} + \eta_{kl} \hat{l}_t) + \epsilon_{t,\tau^k} \quad (16)$$

We will refer to the model closed with the rules (16) and (15) as the recommended model.

## 5. The role of the selected variables in fiscal rules

### 5.1. Welfare comparison

We start the comparison of the baseline model and the recommended model by considering their respective welfare implications. To that end, we follow Schmitt-Grohé and Uribe (2007) and calculate the welfare of the model closed with the simple policy rules relative to the welfare under optimal policy. Welfare costs are measured in steady-state consumption-equivalent units. The steady-state consumption equivalence is approximated using a second-order approximation.<sup>13</sup> We then determine the coefficients in each simple rule which minimize the welfare costs. To ensure theoretically meaningful results, we set bounds on the coefficients. We restrict the coefficient in front of debt to between  $[0, 10]$  for labor income tax rates. Due to the negative steady-state of capital income taxes, its debt coefficient is limited to between  $[-10, 0]$ . Moreover, we assume that all automatic stabilizers have to between  $[-20, 20]$ . The results are shown in Table 1. We find that the model closed with the new feedback rules implies welfare costs which are 35% smaller than those implied by the standard rules or eliminates welfare costs by more than 0.4% consumption-equivalent units.

Fiscal feedback rule	Feedback parameter				Welfare cost in consumption- equivalent units
Baseline model	$\eta_{wb}$ 0.204	$\eta_{kb}$ -1.664	$\eta_{wy}$ -3.788	$\eta_{ky}$ 14.722	1.602%
Recommended model	$\eta_{wb}$ 0.000	$\eta_{kb}$ -3.186	$\eta_{wh}$ -3.697	$\eta_{kl}$ 8.400	1.175 %

Table 1: Welfare costs under different optimized fiscal feedback rules.

### 5.2. Empirical evidence

We estimate the recommended model on the basis of the data, the calibration, and the prior distribution presented in the subsections 3.1 and 3.2. The prior distribution for the coefficients  $\eta_{wh}$  and  $\eta_{kl}$  are chosen to be identical to the prior distribution of the feedback coefficients  $\eta_{wy}$  and  $\eta_{ky}$ . For both coefficients we assume a prior which is normally distributed with mean 0 and standard deviation 0.5. The estimated posterior distributions of the structural parameters are very similar to the estimated posterior distribution in Section 3.2.<sup>14</sup>

<sup>13</sup>More details about the methodology are provided in the online appendix.

<sup>14</sup>As in this section, the model is estimated by running two random walk Metropolis-Hastings chains, each with 1,000,000 parameter vector draws. The first 90% are discarded. The convergence statistics, the posterior plots, and a table summarizing the posterior estimates can be found in the online appendix.

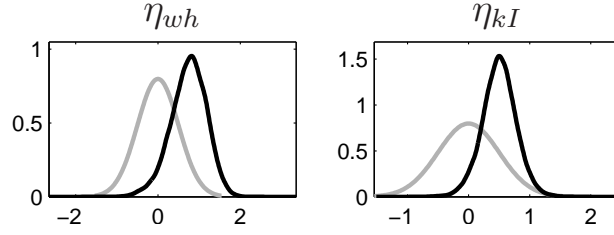


Figure 3: Prior (grey) and posterior (black) distribution of policy feedback parameters.

The prior posterior plots for the new feedback variables are depicted in Figure 3. In contrast to the coefficients in front of output, both feedback coefficients are now identified and estimated as significantly different from zero. The 90% posterior probabilities are [0.03, 1.43] and [0.04, 0.95] for  $\eta_{wh}$  and  $\eta_{kI}$  respectively. The log marginal data densities<sup>15</sup> for the baseline and the recommended model are 4163.99 and 4166.24 respectively. The posterior odds ratio of the baseline model relative to the recommended model is equal to 0.1054. To interpret this ratio we employ the criterion suggested by Jeffreys (1967). According to this, the odds ratio implies substantial evidence against the benchmark model.<sup>16</sup> Given that we have only changed two feedback variables, we consider this finding in addition to the identified coefficients in the data as empirical support in U.S. data for the normatively founded feedback variables.

### 5.3. Policy analysis

In order to investigate the impact of variable selection on policy rules, we consider a policy experiment. We compute dynamic tax multipliers as postulated by Mountford and Uhlig (2009). The present value fiscal multipliers ( $\Delta Y_t / \Delta X_t^i$ ) are computed as

$$\Delta Y_t / \Delta X_t^i(k) = \frac{E_t \sum_{j=0}^k \left( \prod_{i=0}^j R_{t+i}^{b-1} \right) \Delta Y_{t+j}}{E_t \sum_{j=0}^k \left( \prod_{i=0}^j R_{t+i}^{b-1} \right) \Delta X_{t+j}^i}, \quad (17)$$

where  $R^b$  denotes the gross real interest rate,  $k$  the horizon and  $X_t^i$  the tax revenues associated with capital taxes and labor taxes,  $i = [k, w]$ , respectively. Tax multipliers are computed after a tax shock which increases overall tax revenues by 1% in a model without contemporaneous feedback parameters.

	Quarter 1	Quarter 4	Quarter 12	Quarter 20
$\Delta Y / \Delta X^k$				
<i>Baseline model</i>	0.1747	0.3671	0.5763	0.6202
<i>Recommended model</i>	0.1536	0.3282	0.5107	0.5303
$\Delta$ in %	12.07%	10.59%	11.38%	14.49%
$\Delta Y / \Delta X^w$				
<i>Baseline model</i>	0.0324	0.0692	0.1149	0.0894
<i>Recommended model</i>	0.0234	0.0524	0.0831	0.0440
$\Delta$ in %	27.77 %	24.34%	27.60%	50.76%

Table 2: Present value fiscal multiplier for different horizons,  $X_t^i$  denotes the tax revenues with respect to labor and capital income taxes  $i = w, k$ .

Table 2 presents the average present value multipliers for each model based on 500 draws from the corresponding posterior distribution. Similarly to the related literature, we find a small impact capital tax income multiplier (see e.g. Leeper et al., 2010; Zubairy, 2010). In addition, we find an even smaller tax income multiplier for labor income

<sup>15</sup>The marginal data density is computed using the Modified Harmonic Mean estimator as suggested by Geweke (1999).

<sup>16</sup>Jeffreys (1967, p. 432) classifies a value between  $10^{-\frac{1}{2}}$  and  $10^{-1}$  as substantial evidence against the benchmark model (Grade 2).

taxes. In comparison to the literature, this result is related to lower autocorrelation parameters for each tax rate and a smaller Frisch elasticity in the present model. However, there is still a considerable difference between the baseline model and the recommended model. The baseline model systematically overestimates the fiscal multiplier. For the capital income tax rate the difference is between 10% and 14% for different horizons, for the labor income tax rate it is between 24% and 51%.

## 6. Conclusion

The choice of output in income tax feedback rules as variable to capture the state of the economy is neither supported from an empirical point of view nor does a rule that conditions on output approximate optimal policy best. The results of this paper suggest that fiscal rules should be modeled differently. In particular, we show that the variables that have the greatest influence on income tax rates' variance at the optimal allocation are investment and hours worked for capital income tax rates and labor income taxes respectively. Welfare costs measured in consumption equivalence decrease by 35% when the new optimized feedback rules are employed instead of an optimized rule that incorporates output. Finally, we find empirical evidence for our rules in US data and show that the choice of feedback variables matters for policy analysis.

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